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Whole-atom Compton scattering of γ rays, and "magnetic" Compton scattering of circularly polarised synchrotron x-rays

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Abstract: Possible departures of cross sections at small momentum transfer for Compton scattering by high-Z atoms from the predictions of the incoherent-scattering-function approximation are presented and discussed. Studies of "magnetic" Compton scattering of circularly-polarised synchrotron x-rays, emitted at a small angle to the electron orbital plane, from single crystals of ferromagnetic iron are compared with calculations based on the band structure.

Keywords: non-resonant Compton scattering, magnetic Compton scattering

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1. Introduction

Differential cross sections for non-resonant Compton scattering of γ rays and x-rays by high-Z atoms have been determined under conditions of small momentum transfer in order to provide tests of the incoherent-scattering-function approximation. Such studies will be briefly considered in Sec. 2.

The cross sections for "magnetic" Compton scattering of circularly polarised x-rays by ferromagnetic or ferrimagnetic materials show a small, typically < 1%, but very significant contribution depending on the angle between directions of circular polarisation and of the static magnetic field applied in order to orient the magnetic moments. An illustration of such a study at about 60 keV with an iron single crystal performed in collaboration with a group from Warwick University at SRS, Daresbury, U.K. will be presented in Sec. 3.

Conclusions are stated in Sec. 4.

2. Whole-Atom Single-Differential Cross Sections for Compton Scattering

It is convenient to express the atomic Compton scattering cross section in terms of the cross section for scattering by an electron and the incoherent scattering function (ISF) representing the momentum transfer dependent response of the atomic system.

$$\frac{d\sigma^{\text{Compton}}}{d\Omega} = S(x, Z)\,\frac{d\sigma^{\text{KN}}}{d\Omega}, \tag{1}$$

where $S(x, Z)$ has been calculated on the basis of the nonrelativistic A^2 interaction term and other subsidiary assumptions [1], $d\sigma^{KN}/d\Omega$ is the Klein-Nishina prediction for an initially-free and stationary electron, $x = (\sin(\theta/2))/\lambda$ and λ is the wavelength of the incident radiation. The resulting values of S, indicated by S_{WH}, have been tabulated by Hubbell et al. [2] over a large range of x, and by Wang et al. [3] with a finer grid up to $4\ \text{Å}^{-1}$.

A large number of germanium detector results [4-10] for Compton scattering from lead are available at energies far from the binding energy of a lead K shell electron. These are shown in Fig. 1 along with calculated values of S_{WH}/Z. The difficulty of separation of elastic scattering and Compton scattering counts at small values of x is responsible for the absence of data below about $1.2\ \text{Å}^{-1}$. An unexplained scatter of data of Dow et al. [7] is also apparent between $1.7\ \text{Å}^{-1}$ and $4\ \text{Å}^{-1}$. There are only seven data for $x < 3\ \text{Å}^{-1}$, but more than twenty for $3\ \text{Å}^{-1} \le x \le 6\ \text{Å}^{-1}$. For $x > 6\ \text{Å}^{-1}$, almost all the data are consistent with S_{WH} within the experimental errors. But for $x < 5\ \text{Å}^{-1}$, many experimental values with errors of less than $\pm 5\%$ are smaller than S_{WH} by a few %. Two sets of data with errors of about $\pm 10\%$ are in agreement with calculated values of S_{WH} even for $x < 5\ \text{Å}^{-1}$. So it is concluded that additional accurate values are needed particularly for low values of x, if deviations form S_{WH} are to be conclusively established. The small deviations, if any, will indicate small departures from the independent particle approximation (IPA)

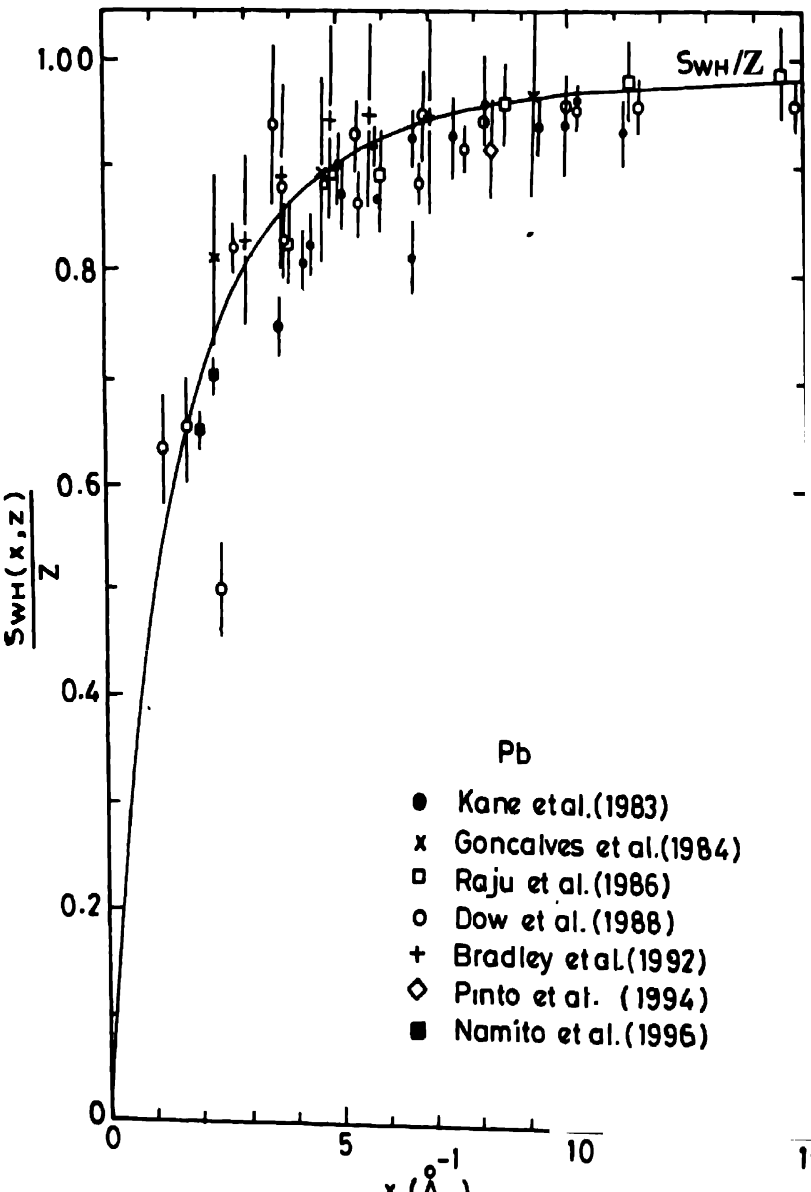

Fig. 1. Values of $[d\sigma^{C}/d\Omega]/[Z(d\sigma^{KN}/d\Omega)]$ determined in the case of lead by various groups are compared with calculations based on the incoherent-scattering-function approximation. See Sec. 2 for additional comments.

and from the underlying assumptions, and possible small differences between non-resonant experimental Compton cross sections and (Raman + Compton) cross sections estimated with the use of S_{WH}. The initial rapid rise of $S(x, Z)/Z$ with x and the slow approach towards unity with further increases in x are certainly confirmed by the different experiments. Data for other targets follow similar trends and are therefore not shown here.

3. Magnetic Compton Scattering

Synchrotron radiation emitted at a small angle (≈ 0.2 milliradian) to the orbital plane has an appreciable degree of circular polarization. The circularly-polarised x-ray beam is monochromatised and allowed to fall on a ferromagnetic or a ferrimagnetic target magnetised by a magnetic field. The latter is reversed every few seconds in order to reverse the direction of magnetisation of the target. The scattering counts for each field direction are stored in separate memories of a computer based multichannel pulse height analyser. At the end of a run lasting typically for a few hours, the difference spectra are obtained and converted into a "magnetic" Compton profile, $J_{\mathrm{mag}}(p_z)$ in a manner analogous to that used in the determination of the well known Compton profile, $J(p_z)$. Here, p_z is the component of the initial electron momentum along the direction of the scattering vector $\boldsymbol{K}$, $\boldsymbol{K} = \boldsymbol{k}_i - \boldsymbol{k}_f$ and $\boldsymbol{k}_i$ (or $\boldsymbol{k}_f$) is the wave vector of the incident (or scattered) radiation.

$$J(p_z) = \iint [n_{\mathrm{up}}(\boldsymbol{p}) + n_{\mathrm{down}}(\boldsymbol{p})]\, dp_x\, dp_y, \tag{2}$$

$$J_{\mathrm{mag}}(p_z) = \iint [n_{\mathrm{up}}(\boldsymbol{p}) - n_{\mathrm{down}}(\boldsymbol{p})]\, dp_x\, dp_y, \tag{3}$$

where n_{up} and n_{down} indicate the densities of "up" and "down" spins, respectively. In iron, the orbital angular momentum is quenched and the magnetic-moment-dependent scattering is mainly due to spin. Further, the ratio of the number of unpaired electrons to the total number is smaller than 0.1. At photon energies of the order of 50 keV ($\approx 0.1\ \mathrm{mc}^2$), the "magnetic" scattering amplitude is of the order of 0.1 times the charge scattering amplitude. Thus the counting-rate difference Δ for opposite directions of the externally-applied magnetic field is typically less than 1% of the average measured rate and is given by the following relation.

$$\Delta(h\nu_f) \propto P_c\,(1 - \cos\theta)\, \boldsymbol{S} \cdot (\boldsymbol{k}_i \cos\theta + \boldsymbol{k}_f)\, J_{\mathrm{mag}}(p_z)\ , \tag{4}$$

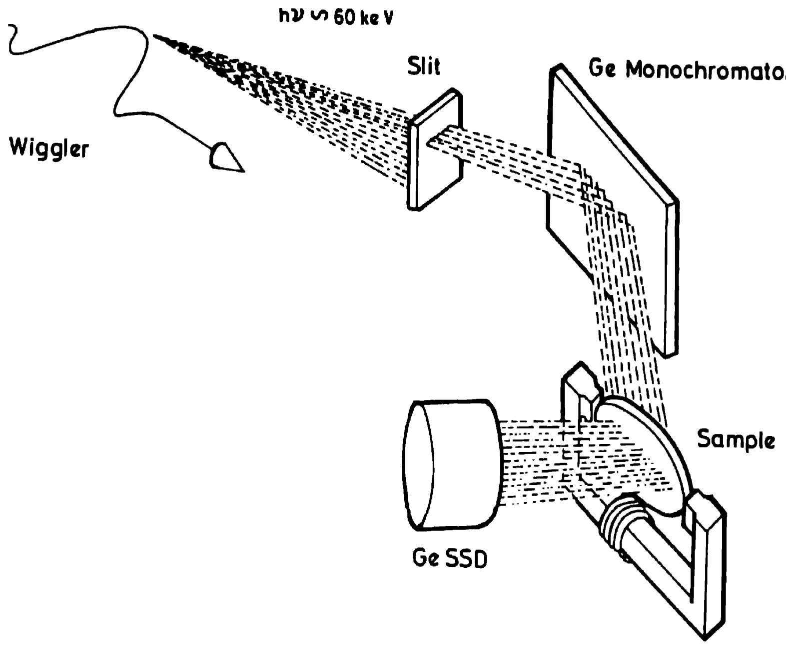

Fig. 2. A schematic experimental arrangement [11] for a study of "magnetic" Compton scattering.

where P_c is the circular polarization of the x-ray beam and

$$\frac{p_z}{mc} = \frac{\nu_f - \nu_i + h\nu_i\nu_f(1-\cos\theta)/(mc^2)}{(\nu_i^2 + \nu_f^2 - 2\nu_i\nu_f\cos\theta)^{1/2}} . \tag{5}$$

The experimental arrangement [11] is shown schematically in Fig. 2. The data obtained for the three major crystallographic directions of a single crystal of iron [12] are shown in Fig. 3. The full lines indicate calculations of Wakoh [13] based on the augmented-plane-wave (APW) method. Unlike $J(p_z)$, $J_{mag}(p_z)$ has a minimum at low values of p_z. The minimum is deepest in the case of the [111] direction. The dips at low momentum arise due to the *s-p* like conduction electrons having opposite polarization to those in the more localized 3*d* states. This interpretation is supported by spin-polarized positron annihilation experiments and by polarized-neutron diffraction experiments. Differences between J_{mag} in the different directions have been calculated on the basis of band theory by Callaway and coworkers [14] and by Kubo and coworkers [15]. Data of higher precision obtained recently for a large number

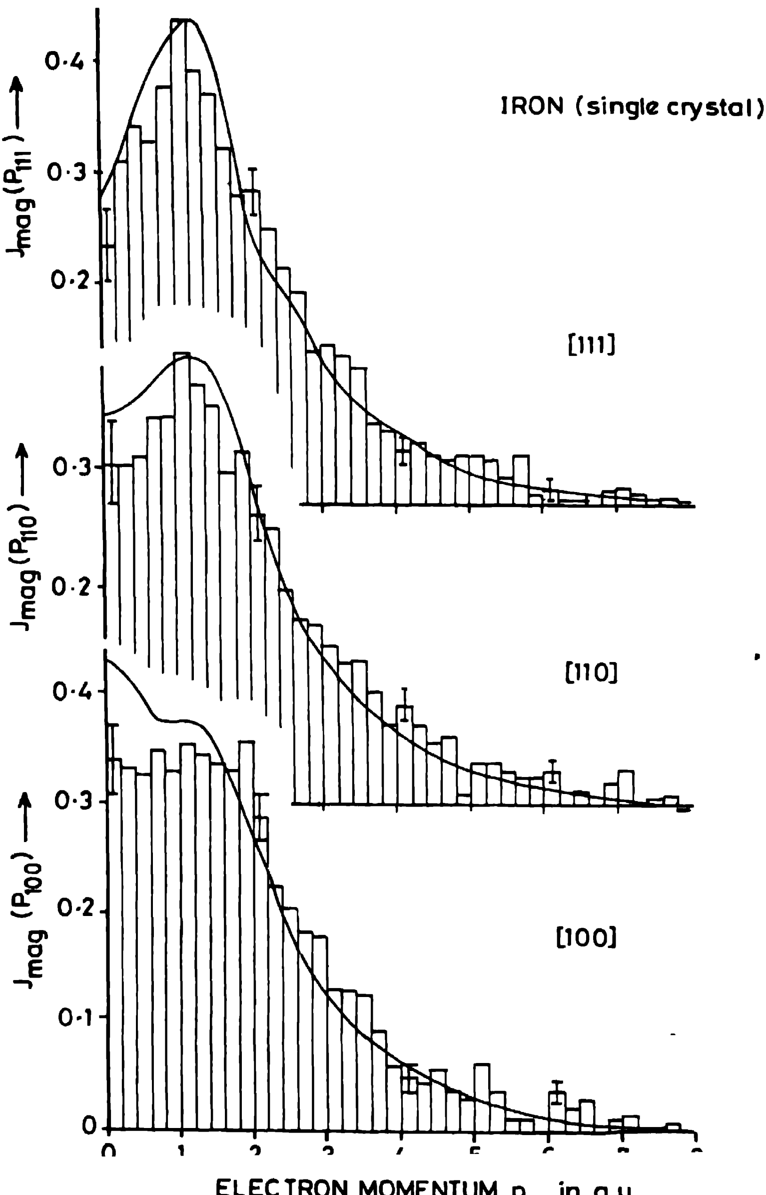

Fig. 3. "Magnetic" Compton profiles determined in the case of an iron single crystal for three crystallographic directions. The solid lines indicate calculated profiles. See Sec. 3 for more details.

of crystallographic directions [16] confirm the larger negative polarization of conduction electrons than that calculated in certain directions. Further details are not discussed in this brief report.

4. Conclusions

Differential cross sections for whole-atom Compton scattering for small values of momentum transfer ($x \leq 6$ Å^{-1}) are found to be a few % smaller than the values predicted by the incoherent-scattering-function approximation, indicating the need for further experimental and theoretical studies. The role of "magnetic" scattering in elucidating the band structure of single-crystal iron and other ferromagnetic or ferrimagnetic materials has been demonstrated.

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